Stochastic population modeling

Ola Diserud
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NINA
Norwegian Institute for Nature Research

3.10 Quasi stationary distribution

Process not stationary $\Rightarrow E(T) = \int_a^\infty G(n, n_0)\, dn < \infty$

Scale the Green function:

$$q(n, n_0) = \frac{G(n, n_0)}{\int_a^b G(z, n_0)\, dz}$$

This is the quasi stationary distribution

Fig. 3.6

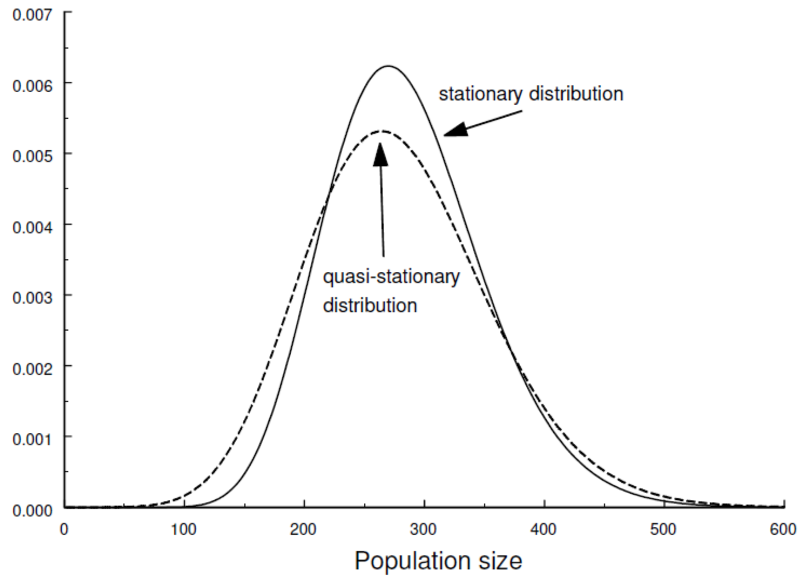

Figure 3.6: The stationary distribution, the gamma distribution, for the logistic model with absorbing barrier at zero and parameters $r = 0.1$, $K = 300$ and $\sigma_e^2 = 0.01$ (solid line) together with the quasi-stationary distribution with $N_0 = K$ obtained by adding demographic variance $\sigma_d^2 = 1$ to the model.

PVA – Population Viability Analysis

Viability = Probability that a population goes extinct within a time interval.

$$P\left(T > t_v\right) > 1 - \alpha_v$$

Full PVA:

1. Set up a model
2. Estimate parameters from time series & demographic info if available.
3. Evaluate uncertainty in estimates & possible changes in parameters.

3.11.2 Exponential approximation

Example:

E(T)=2000 yrs

$$P\left(T < 500\right) \approx 1 - e^{-500/2000} = 0.2212$$

$$P\left(T < 200\right) \approx 0.0952$$

$$P\left(T < 100\right) \approx 0.0488$$

Fig. 3.8, p. 88

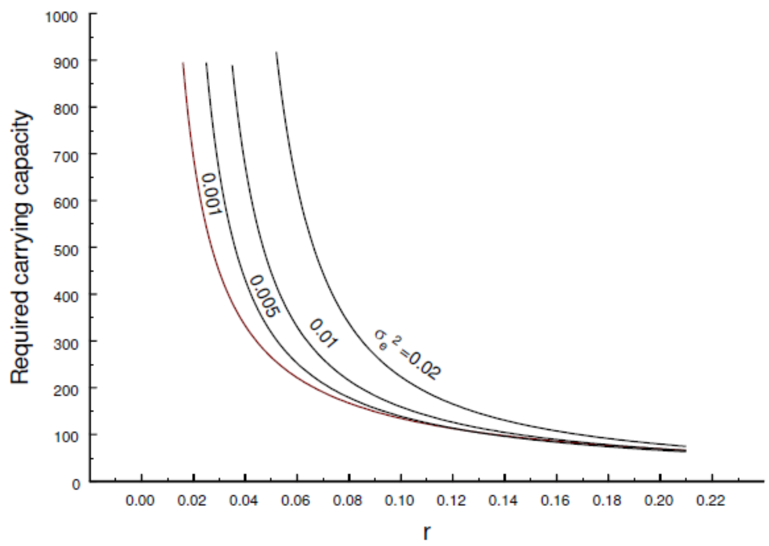

Figure 3.8: The carrying capacity required to obtain a probability of extinction at most 0.1 within a 100 years period for the logistic type of model. There is one graph for four different values of the environmental variance. The demographic variance is 1 and the initial population size is at the carrying capacity.

3.12 Autocorrelations

Gompertz' model:

$$\mu_N(n) = r_1 n\left[1 - \frac{\ln n}{\ln K}\right]$$

$$X = \ln N \Rightarrow \mu_X(x) = r_1\left[1 - \frac{x}{\ln K}\right] - \frac{1}{2}\sigma_e^2$$

$$\Rightarrow \rho_X(h) = e^{-\frac{r_1}{\ln K}h}$$

Theta-logistic model: $\theta \neq 0, \sigma_d^2 = 0$

$$\mu_N(n) = r_1 n\left[1 - \frac{n^\theta - 1}{K^\theta - 1}\right]$$

$$Y = N^{-\theta} \Rightarrow \mu_Y(y) = \frac{r^\theta}{K^\theta} - y\theta\left[s - \frac{1}{2}\theta\sigma_e^2\right]$$

$$\Rightarrow \rho_Y(h) = e^{-\theta\left(s - \frac{1}{2}\theta\sigma_e^2\right)h}$$

Fig 3.12, p. 100

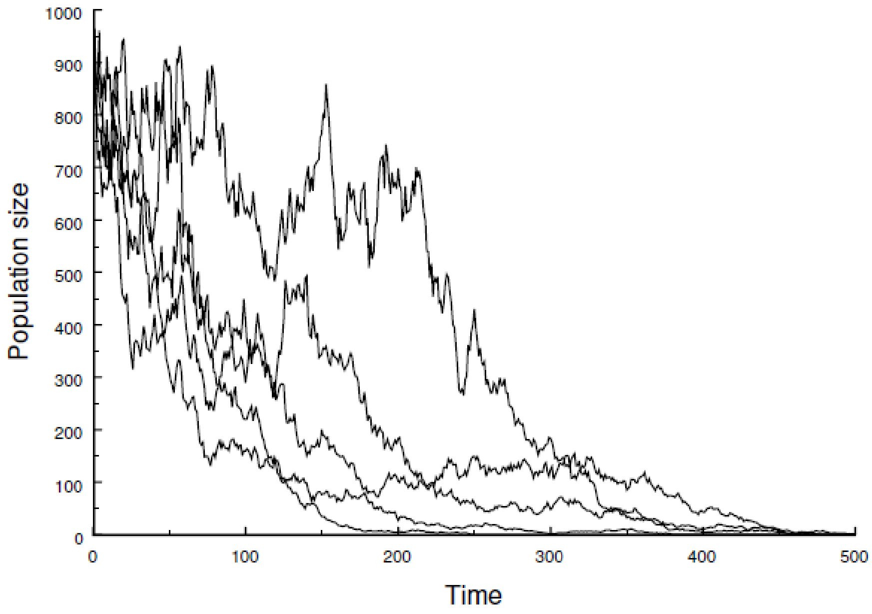

Figure 3.12: Simulations of the final decline from the carrying capacity to extinction in the logistic model with parameters $r = 0.2$, $K = 1000$, $\sigma_d^2 = 1$ and $\sigma_e^2 = 0.01$.

3.15 Autocorrelated noise

$N_{t+1} = \Lambda_t N_t$; Λ_t and Λ_{t+h} can be correlated

$$\Rightarrow \rho(h) = \rho(-h) = corr(\ln\Lambda_t, \ln\Lambda_{t+h}) = corr(S_t, S_{t+h})$$

where $\sigma^2 = Var(S_t)$

$$Var(X_t \mid x_0) = Var(S_0 + S_1 + S_2 + \ldots + S_{t-1})$$
$$= t\sigma^2 + 2(t-1)\rho(1)\sigma^2 + 2(t-2)\rho(2)\sigma^2 + \ldots + 2\rho(t-1)\sigma^2$$
$$= \sigma^2 t \sum_{i=-(t-1)}^{t-1} \rho(i) - 2\sigma^2 \sum_{i=1}^{t-1} i\rho(i)$$

3.15 Autocorrelated noise

$$\lim_{t\to\infty}\left(\frac{Var\left(X_t \mid x_0\right)}{t}\right)\to\sigma^2\sum_{-\infty}^{\infty}\rho\left(i\right)$$

For a process with autocorrelated noise, we get the correct expectation and variance for $X_t \mid x_0$ if we approximate it with a process with white noise with variance $\sigma^2\sum_{-\infty}^{\infty}\rho\left(i\right)=\sigma^2\left[1+2\sum_{i=1}^{\infty}\rho\left(i\right)\right]$

Fig 3.14, p. 106

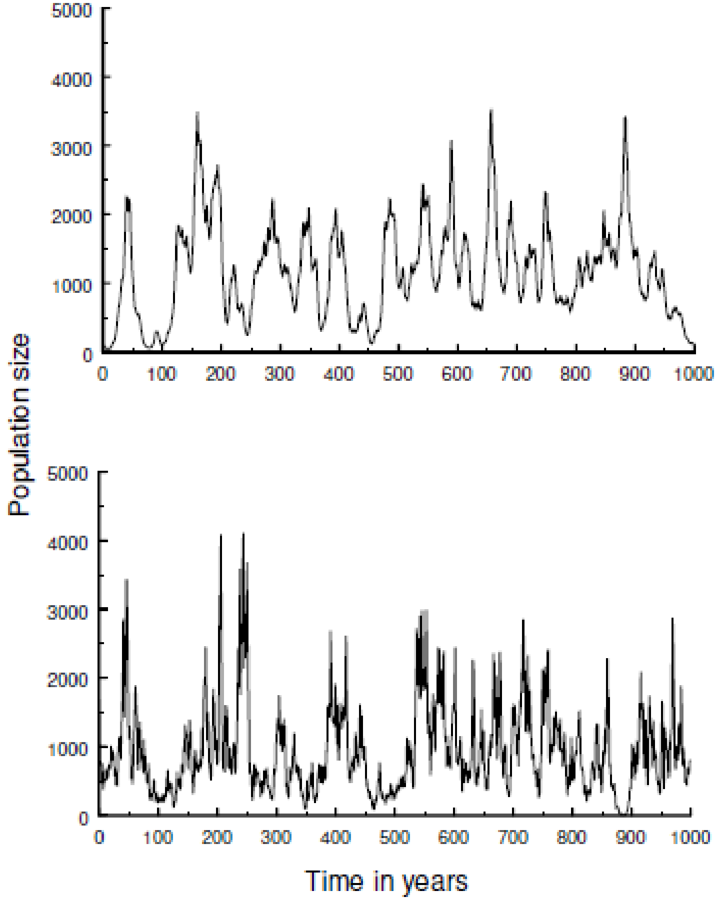

Figure 3.14: The same model as in the lower panel of Fig.3.13 with $\gamma = 0.8$ but showing the processes separately for 1000 years. The upper panel is the process with autocorrelated noise and the lower panel is the diffusion approximation.

Fig. 3.15, p. 107

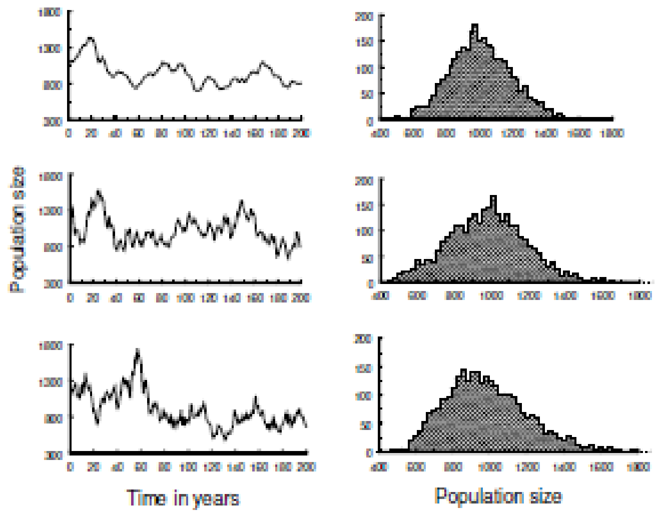

Figure 3.15: Simulations of the continuous processes (left panels) of the logistic type during 200 years with $r = 0.1$, $K = 1000$, and $\sigma_e^2 = 0.01$ with noise process Z_t of the Ornstein-Uhlenbeck type shown together with histograms (right panels) of the stationary distributions based on 30000 years. In the upper panel $\beta = 0.3$, in the middle $\beta = 2$ and in the lower $\beta = 100$. In all simulations $\omega^2 = \beta^2$. In the process with $\beta = 100$ the noise Z_t is practically white noise.